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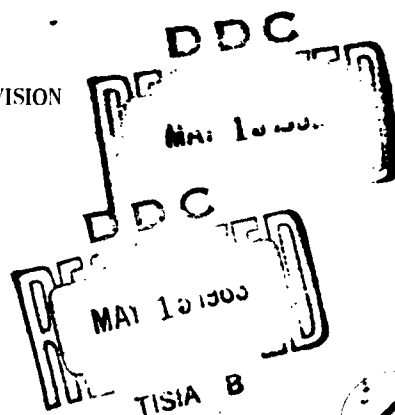
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# Self-Contained Explicit Guidance Equations for Ballistic Missiles

MARCH 1963

*Prepared by*  
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*Prepared for* COMMANDER  
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AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
*Norton Air Force Base, California*



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**Report No.  
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FOR BALLISTIC MISSILES**

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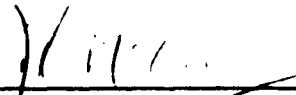
SELF-CONTAINED EXPLICIT GUIDANCE EQUATIONS  
FOR BALLISTIC MISSILES

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## **ABSTRACT**

**This document outlines a guidance equation mechanization for use with ballistic missiles. This technique does not require pre-launch targeting computations and is therefore especially suitable for mobile weapons systems.**

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## SECTION I

### INTRODUCTION

The development of a guidance technique suitable for use with any ballistic missile is presented in this document. This technique is particularly suitable for weapon systems requiring maximum flexibility (mobile systems requiring a minimum operational targeting time). While an example of the basic equations required for mechanization is given, relatively routine aspects of the mechanization (such as data reduction and first stage and vernier guidance) are not discussed. This mechanization is obviously more complicated than that which would be required if the equations were not self contained (i.e., if targeting operations are carried out at the launch site).

The equations of motion which have been used for the explicit solution on the free flight trajectory are derived from the assumptions that the earth's gravitational field varies with the inverse square law and that there is no atmosphere (see Section 3). The inadequacy of this assumption can be compensated for by empirically determined target offsets, but a considerably superior method which does not require any targeting operations at the operational site is developed in Section 4.

The only inputs required by the guidance equations are latitude, longitude, and altitude (above sea level) of the launcher and target. The guidance equations then compute all other quantities required, including launch azimuth. The guidance inputs need to be specified sufficiently prior to launch to permit the launch azimuth to be implemented (computation time is negligible). With the possible exception of hardware requirements there is no other reason for a delay between the receipt of information regarding target and launcher locations and launch of the vehicle.



Simulation has shown that the one  $\sigma$  miss distance due to the inaccuracies in this technique can easily be reduced to 200 feet. Considerably better accuracy may be obtained if empirical coefficients are determined carefully. The above error does not include the effects of gravitational anomalies, hardware inaccuracies or measurement errors.

## SECTION 2

### DEFINITIONS

The following definitions are given to aid in understanding the discussion which follows:

Explicit Guidance is a generic term for the class of guidance equations which are a direct solution of equations of motion. The equations of motion which are used may, or may not, represent reality with sufficient accuracy to satisfy mission objective; there is, however, a restriction in the latter case that the inaccuracy (which is compensated for by empirical or semi-empirical methods) must be extremely small.

Simulation is a computer (usually high-speed digital) program which is an accurate representation of the earth, atmosphere, missile engine and autopilot, etc., as well as the guidance equations. This simulation then "flies" the missile and determines performance capability, workability of the guidance equations, etc.

Targeting is the utilization of a simulation to define and verify any empirical constants that may be required by the guidance equations. For many problems efficient use of the simulation to obtain the empirical constants requires the use of auxiliary computer programs; this kind of effort is, of course, part of targeting. The word targeting is sometimes applied to operations (carried out at the operational site) which utilize the empirical constants obtained by the process defined above as targeting.

Basic Inertial Coordinates  $\xi$ ,  $\eta$ ,  $\zeta$  are earth centered with  $\zeta$  passing through the north pole. These coordinates and their derivatives are scaled so that the gravitational constant multiplied by the mass of the earth (GM) is unity. At some time during the pre-launch operation, the gyros are uncaged. The inertial coordinate system is defined at this instant and the quantity  $T_L$  represents the time interval which has elapsed since this instant. These coordinates would not necessarily be used in an actual mechanization, particularly for an inertial system.

## SECTION 3

### A KEPLERIAN SOLUTION

#### Coordinate Systems

It is necessary to relate the position and velocity of the missile and the target by means of an inertial coordinate frame. Once this has been done, any appropriate criteria can be used to develop expressions for pitch and yaw velocities. Criteria which have been used in the past and have proven to be very satisfactory are outlined below.

The missile, target and geocentroid define a plane. The yaw component of velocity is the component of missile velocity normal to this plane and the pitch component of velocity is the component of missile velocity in this plane and parallel to the radius from the geocentroid to the missile. Pitch angles are measured positive up from the normal to this radius. (See Figure 1 and Appendix A.)

The subscript D is used to denote burnout conditions. Adequate estimation of  $\psi_D$  is straightforward (see Appendix A).

#### Orbital Parameters

The value of  $\Gamma_T$  is defined as a compromise between minimum re-entry dispersions, re-entry heating problems, and performance. Actually the numerical coefficients in the definition of  $\Gamma_T$  (Appendix A) can be chosen to provide a good approximation to any reasonable functional relationship for  $\Gamma_T$ . The desirability of steering to a given  $\Gamma_T$  is related to re-entry effects and is discussed in Section 4.

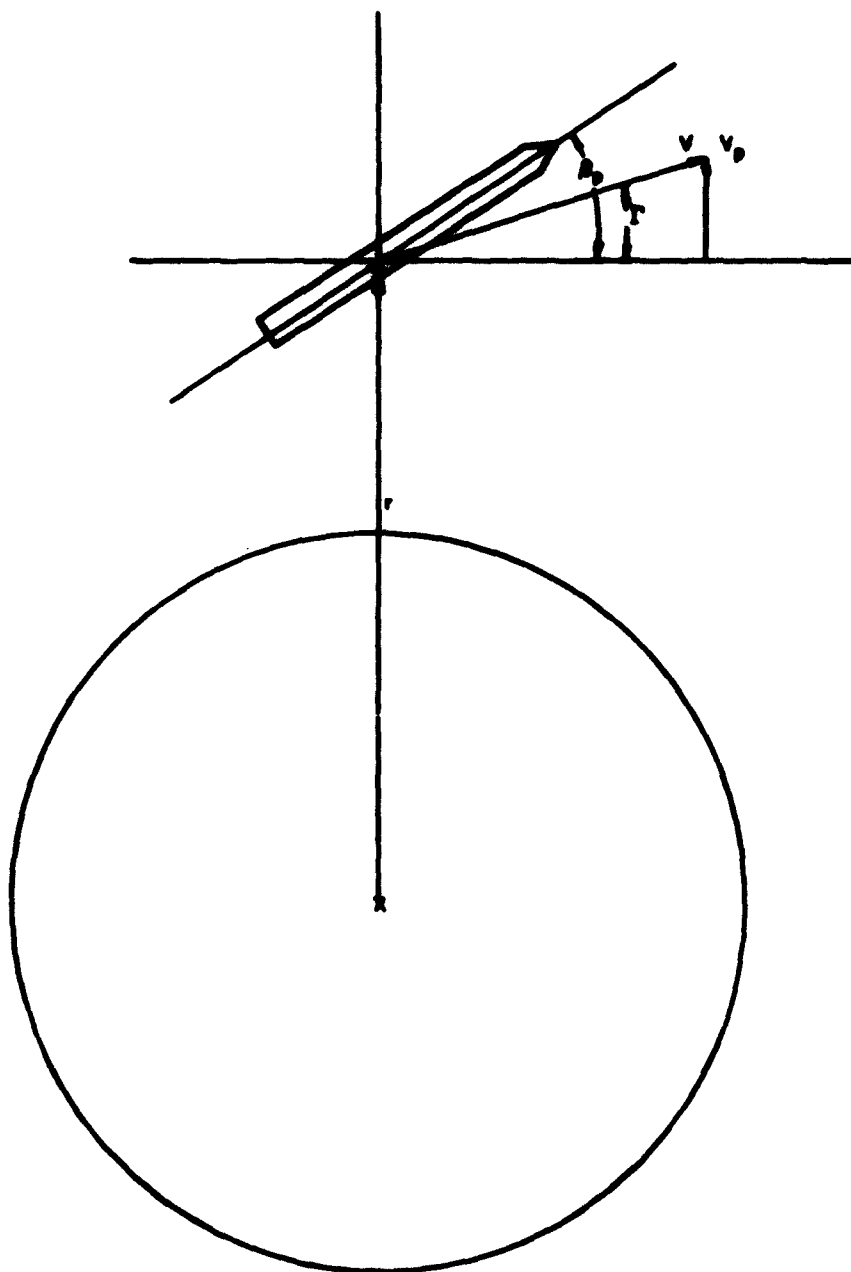


Figure 1. Definition of Coordinates

The expression for the velocity necessary to travel from any point in space to any other point (assuming no atmosphere and an inverse square gravitational field) can be derived from Figure 2 and the well known orbit equation

$$V^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) = \frac{2}{r} - \frac{1}{a} \quad (1)$$

where  $a$  is the semi-major axis of the orbit. From Figure 2

$$\begin{aligned} (2a - r_2)^2 &= [r_2 \sin \psi - (2a - r_1) \sin 2\Gamma_1]^2 \\ &\quad + [r_1 - (2a - r_1) \cos 2\Gamma_1 - r_2 \cos \psi]^2 \end{aligned}$$

or

$$a = \frac{r_1}{2} \left[ 1 + \frac{r_2(1 - \cos \psi)}{r_1(1 + \cos 2\Gamma_1) - r_2[1 + \cos(\psi + 2\Gamma_1)]} \right] \quad (2)$$

Combining Equations 1 and 2

$$r_1 V_1^2 = \frac{2}{r_1} \left( 1 - \frac{r_1}{2a} \right) = \frac{2(1 - \cos \psi)}{\frac{r_1}{r_2}(1 + \cos 2\Gamma_1) - \cos \psi - \cos(\psi + 2\Gamma_1)} \quad (3)$$

Defining

$$\delta \equiv r V^2$$



and choosing  $r_1$  as the target and  $r_2$  as burnout

$$\delta_T = \frac{2(1 - \cos \psi_1)}{\frac{r_T}{r_D} (1 + \cos 2\Gamma_T) - \cos \psi_D - \cos (\psi_D + 2\Gamma_T)} \quad (4)$$

and

$$v_T = \sqrt{\frac{\delta_T}{r_T}} \quad (5)$$

from Equation 1

$$a = \frac{r_T}{2 - \delta_T} \quad (6)$$

From orbital kinematics (Reference 2)

$$\epsilon \sin E = \sqrt{\delta(2 - \delta)} \sin \Gamma \quad (7)$$

$$\epsilon \cos E = \delta - 1 \quad (8)$$

$$T_{FF} = a^{3/2} [E_2 - E_1 - \epsilon \sin E_2 + \epsilon \sin E_1] \quad (9)$$

Then

$$\begin{aligned} \epsilon &= \sqrt{(\delta_T - 1)^2 + \delta_T(2 - \delta_T) \sin^2 \Gamma_T} \\ &= \sqrt{1 - \delta_T(2 - \delta_T) \cos^2 \Gamma_T} \end{aligned} \quad (10)$$

and

$$\delta_D = 2 - \frac{r_D}{a}$$

$$v_D = \sqrt{\frac{\delta_D}{r_D}} .$$

An expression for  $\Gamma_D$  may be found from Equation 3 as follows:

$$\frac{2}{\delta_D} (1 - \cos \psi_D) = \frac{r_D}{r_T} (1 + \cos 2\Gamma_D) - \cos \psi_D - \cos (\psi_D + 2\Gamma_D)$$

since

$$\cos (\psi_D + 2\Gamma_D) = \cos \psi_D \cos 2\Gamma_D - \sin \psi_D \sin 2\Gamma_D .$$

Then

$$\sin 2\Gamma_D = Q_2 - Q_1 (1 + \cos 2\Gamma_D) \quad (11)$$

where

$$Q_1 = \frac{\frac{r_D}{r_T} - \cos \psi_D}{\sin \psi_D}$$

$$Q_2 = \frac{2(1 - \cos \psi_D)}{\delta_D \sin \psi_D} . \quad (12)$$



Squaring Equation 11

$$\sin^2 2\Gamma_D = (Q_2 - Q_1)^2 - 2(Q_2 - Q_1)Q_1 \cos 2\Gamma_D + Q_1^2 \cos^2 2\Gamma_D$$

or

$$(1 + Q_1^2) \cos^2 2\Gamma_D - [2(Q_2 - Q_1)Q_1] \cos 2\Gamma + [(Q_2 - Q_1)^2 - 1] = 0$$

Then

$$\begin{aligned} \cos 2\Gamma_D &= \frac{2(Q_2 - Q_1)Q_1 \pm \sqrt{4Q_1^2(Q_2 - Q_1)^2 - 4(1 + Q_1^2)[(Q_2 - Q_1)^2 - 1]}}{2(1 + Q_1^2)} \\ &= \frac{Q_1(Q_2 - Q_1) \pm \sqrt{1 - Q_2^2 + 2Q_1Q_2}}{1 + Q_1^2} \end{aligned}$$

Obviously the sign of the radical depends on whether a lofted or shallow trajectory is used (the radical vanishes at minimum energy). Since a slightly lofted trajectory will be used (see Section 4)

$$\cos 2\Gamma_D = \frac{Q_1(Q_2 - Q_1) - \sqrt{1 - Q_2^2 + 2Q_1Q_2}}{1 + Q_1^2} \quad (13)$$

A convenient specialized solution of Kepler's equation (Eq. 9) for ballistic missile trajectories can be developed as follows:

$$e \cos E = 1 - \frac{r}{a}$$

Obviously  $E = 0$  at perigee, and since  $a$  is less than the radius of the earth  $\pi/2 < E < 3\pi/2$  throughout the trajectory. However if

$$E_T = \cos^{-1} \left[ \frac{\frac{r_T}{a} - 1}{G} \right] \quad (14)$$

$$E_D = \cos^{-1} \left[ \frac{\frac{r_D}{a} - 1}{G} \right]$$

are evaluated in the first quadrant then Equation 9 becomes

$$T_{FF} = a^{3/2} [E_T + E_D + e(\sin E_D + \sin E_T)] \quad (15)$$

## SECTION 4

### NON-KEPLERIAN EFFECTS

#### Introduction

The two-body point mass central force field problem can always be reduced to an equivalent one-body problem (Reference 2). For an inverse square force field this becomes the "Keplerian problem," and more or less complete analytic treatments have been known for centuries. For practical purposes on a ballistic missile trajectory the solution to the Keplerian problem is in error due only to:

- a. Atmospheric forces at re-entry
- b. The fact that the earth's gravitational field is not exactly inverse square, but has other latitude dependent terms
- c. Local gravitational anomalies, and
- d. Solar and lunar gravity.

Fortunately these effects are quite small.

#### Atmospheric Effects at Re-entry

For practical purposes the only non-trivial complications arising from inclusion of a target altitude are due to atmospheric forces at re-entry. The principal result of these atmospheric forces is to:

- a. Increase the time required to reach the target altitude
- b. Cause the hit point to be less far "down range" in the orbit plane, and
- c. Impact displacement due to atmospheric (earth) rotation.

It is convenient to combine (a) and (c) to obtain an "effective" time differential (which is quite small). Since  $\Gamma_T$  has been defined as a function of

$V_T$  (see Fig. A-3) the two important re-entry variables ( $V$  and  $\Gamma$ ) have been reduced to a one parameter family. With this simplification it is possible to compensate for re-entry effects (for altitudes of up to one mile) with an rms error of about 100 ft with the following empirical equations:

$$\psi_{RE} = K_0^{14} + K_1^{14} V_T + K_2^{14} V_T^2 + K_3^{14} V_T^3 + K_4^{14} V_T^4 + (K_5^{14} + K_6^{14} V_T + K_7^{14} V_T^2) h_T + (K_8^{14} + K_9^{14} V_T + K_{10}^{14} V_T^2) h_T^2 \quad (16)$$

$$T_{RE} = K_{20}^{14} + K_{21}^{14} h_T + K_{22}^{14} V_T \quad (17)$$

Equations 16 and 17 are terms in  $\psi_{MT}$  and  $T_F$  (see Appendix A).

It is of course desirable to have  $\psi_{RE}$  and  $T_{RE}$  as small as possible which would require a very lofted trajectory. However when this effect is balanced against the requirements of heating and trajectory sensitivity (miss coefficients) the resulting compromise is a family of trajectories slightly more lofted than the "minimum energy" family.

#### Oblate Potential

The earth's potential field is usually represented as

$$V = - \frac{GM_m}{r_o} \left[ \frac{r_o}{r} + J \frac{r_o^3}{r^3} \left( \frac{1}{3} - \sin^2 \theta \right) + D \frac{r_o^5}{r^5} f(\sin^2 \theta) + \dots \right]$$

where  $J$  and  $D$  are constants. The Keplerian analysis assumes that  $J$ ,  $D$ , and all higher order terms vanish, an assumption that would give intolerable error if not compensated for. The basis for this oblate potential compensation is the perturbation development by Wheelon in Ref. 3.

The D term is about one tenth percent of the size of J term (not one percent as stated in Ref. 3) and the higher order terms are even smaller, and since the error in the J perturbation is larger than this (due to neglect of the second order terms in the perturbation development) primary consideration will be given to the first order perturbation of the J term. All residual errors will then be lumped together and can be treated empirically.

Wheelon develops (Ref. 3 Eqs. 65 - 69) the following expression for cross range miss (in present notation):

$$\delta L = \frac{-2Jr_T}{\delta_D \cos^2 \Gamma_D} \sin A_L \cos \theta_L \left[ \sin \theta_L \int_0^\psi r^2(X) \int_0^x \frac{\cos y}{r^2(y)} dy \right. \\ \left. + \cos A_z \cos \theta_L \int_0^\psi r^2(X) \int_0^x \frac{\sin y}{r^2(y)} dy \right] \quad (18)$$

Wheelon now sets r constant and integrates. A more general expression is

$$\delta L = \frac{2Jr_T}{\delta_D \cos^2 \Gamma_D} \sin A_z \cos \theta_L [\sin \theta_L (1 - \cos \psi_D) f_1(\psi_D) \\ + \cos A_z \cos \theta_L (\psi_D - \sin \psi_D) f_2(\psi_D)] \quad (19)$$

[Wheelon has  $f_1(\psi_D) = f_2(\psi_D) = 1$ ]. Expressions for  $f_1$  and  $f_2$  (which will vary slightly depending on the trajectory used) can now be developed empirically. For ICBM trajectories of tactical interest the simple expression

$$f_1 = f_2 = (K_0 + K_1 \theta_L) + (K_2 + K_3 \theta_L) \psi^* \quad (20)$$

is very satisfactory.

Since  $\delta L$  is effectively a change in the desired yaw plane

$$v_{yD} = \frac{-V_D \cos \Gamma_D}{r_T \sin \psi_D} \delta L \quad (21)$$

An expression similar to Eq. 19 can be developed from Wheelon's analysis:

$$\delta R = \frac{-2Jr_T(1 - \cos \psi_D)f_3}{\delta_D^2 \sin 2\Gamma_D \cos^2 \Gamma_D} \left[ f_3 \delta_D \cos^2 \Gamma_D (1 - Q_{12} + f_4) + \frac{2}{3} (Q_{12} - 3 \sin^2 \theta_L) \right] \quad (22)$$

where

$$Q_{12} = \sin^2 \theta_L (2 + \cos \psi_D) + 2 \sin \theta_L \cos \theta_L \cos A_z \sin \psi_D \\ + \cos^2 A_z \cos^2 \theta_L (1 - \cos \psi_D).$$

The functional form of  $f_3$  is chosen to be

$$f_3 = \frac{1}{1 + \left| \frac{a}{r_T} (1 + \epsilon) - 1 \right| K_0^{16}} \quad (23)$$

The function  $f_4$  arises from a  $\frac{dr}{d\psi}$  term and thus no neat semi-empirical relationship can be found by a constant  $r$  integration. A satisfactory representation for ICBM trajectories of tactical interest has been found to be

$$f_4 = \left( \sin^2 \theta_L - \cos^2 \theta_L \cos^2 A_z \right) (K_0^{20} + K_1^{20} \psi^* + K_2^{20} \psi^{*2}) \\ - 2 \sin \theta_L \cos \theta_L \cos A_z (K_0^{21} + K_1^{21} \psi^* + K_2^{21} \psi^{*2}) \quad (24)$$

Equations 22, 23 and 24 can be combined with the relationship

$$J = - \frac{\delta R}{r_T} \quad (25)$$

To obtain a correction to  $\psi_{MT}$  (see Fig. A-2).

The oblate potential also changes the time of free flight from that given by the Keplerian solution. This variation can be approximated empirically by the expression

$$T_J = (K_0 + K_1 \theta_L) + (K_2 + K_3 \theta_L) \psi^* + (K_4 + K_5 \theta_L) \psi^{*2} - \psi^{*2} (K_6 + K_7 \theta_L) \sin |2A_Z| \quad (26)$$

#### Local Gravitational Anomalies

Local gravitational anomalies perturb the trajectory not only in free flight but in powered flight as well. Unfortunately these anomalies have not been mapped completely enough to allow a definitive analysis of their effect on trajectories of general interest. Once this mapping becomes available there is no conceptual reason why these effects could not be included in an empirical way, although the amount of information that would have to be stored might be prohibitive.

### Solar and Lunar Gravity

Solar and lunar gravitational fields perturb the trajectory whenever the vehicle and the geocentroid are not equidistant from these bodies. The perturbing acceleration is

$$A_{\text{pert}} = \frac{GM}{R_1^2} - \frac{GM}{R_2^2} = \frac{GM}{R_1^2} \left( 1 - \frac{R_1^2}{R_2^2} \right) \approx \frac{GM}{R_1^2} \frac{2 \delta R}{R_1}$$

where

$$\delta R = R_2 - R_1 \lesssim \text{radius of earth}$$

For the sun

$$a_{\text{pert sun}} = 2\omega_e^2 \delta R$$

where

$$\omega_e \approx 1^\circ/\text{day} \approx 2(10^{-7}) \frac{\text{rad}}{\text{sec}}$$

$$\delta R \approx 2(10^7) \text{ ft}$$

so

$$a_{\text{pert sun}} \approx 1.6(10^{-6}) \text{ ft/sec}^2$$



Since

$$S_{\text{pert}_{\text{sun}}} \leq \frac{1}{2} a_{\text{pert}_{\text{sun}}} T_F^2 = 0.8(10^{-6})(2000)^2 \approx 3 \text{ feet}$$

then

$$R_{\text{pert}_{\text{sun}}} < \frac{S_{\text{pert}_{\text{sun}}}}{\sin RE} \approx 7 \text{ feet}$$

It can similarly be shown that  $R_{\text{pert}_{\text{moon}}} < 1$  foot, and thus effects of solar and lunar gravity can be ignored.

#### Residual Errors

After all the preceding empirical and semi-empirical corrections are made there are still residual errors. These errors are from three primary sources:

a. Inaccuracies in the Approximating Functions

Inevitable approximations exist in any empirical technique. This is especially true for the functions given in this section, since they are primarily a feasibility demonstration and could certainly be improved.

b. Neglected Effects

All terms except the J term in the potential expansion are examples.

c. Neglect of the Inter-Dependence of the Various Effects

Trajectory perturbations due to oblateness obviously have an effect on re-entry perturbations and vice versa. Likewise trajectory dispersions due to non-nominal missile parameters affect the oblateness correction (but not re-entry since  $\Gamma_T$  is a function of  $V_T$ ). It would be possible to correct for these effects empirically, although they are quite small.

A representative plot of these residual errors as determined from simulation is given in Figure 3. It would of course be possible to tabulate these residual errors and utilize them to obtain extreme accuracy if a small amount of prelaunch computation is acceptable.

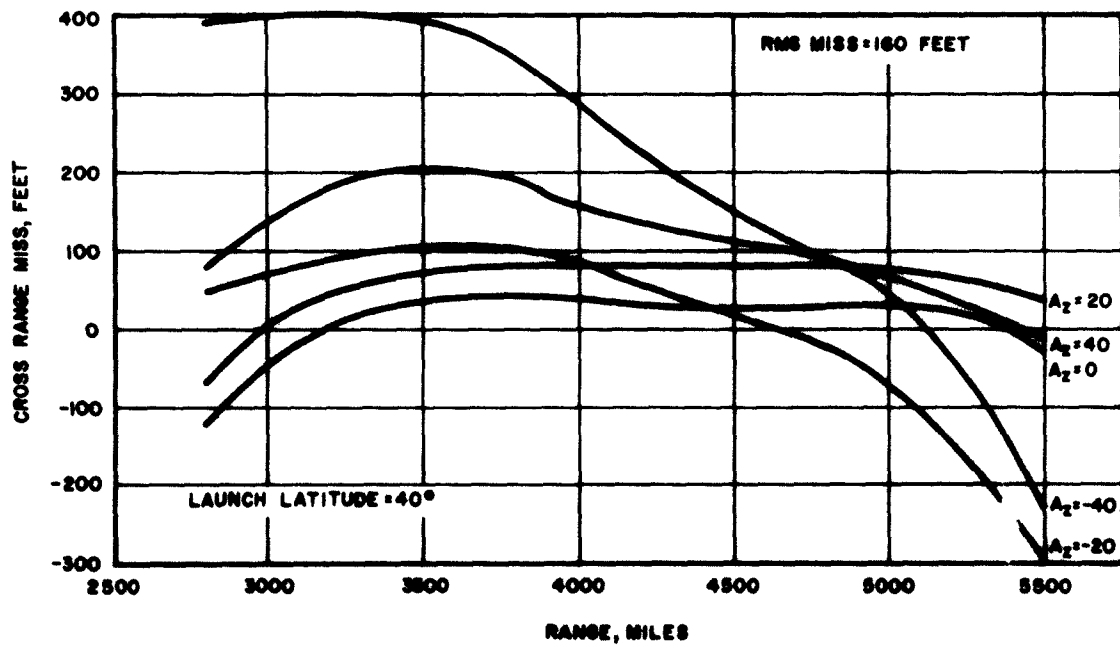
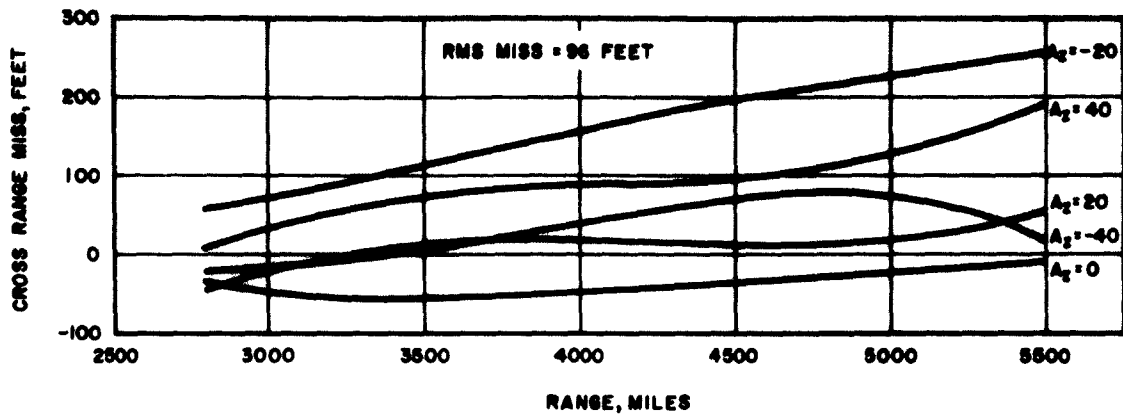


Figure 3. Representative Plot of Residual Errors Determined from Simulation

## APPENDIX A

### GUIDANCE EQUATIONS

Equations which can be used to generate thrust termination and steering commands are discussed in detail in Reference 1. The derivation is lengthy and will not be given here. These equations are written without reference to any nominal missile, which means that dispersions due to missile parameter variations vanish except for secondary variations in the non-Keplerian effects. The vehicle is steered to obtain  $V = V_D$ ,  $V_y = V_{yD}$  and  $\Gamma = \Gamma_D$  at burnout, where the desired quantities are defined from Sections 3 and 4.

An illustrative example of the essential elements of a guidance equation mechanization is given in the following figures. The symbol  $\tau$  represents the duration of the computing cycle. Any value of  $\tau$  between 0.5 and 1.5 seconds will work equally well for practical purposes.

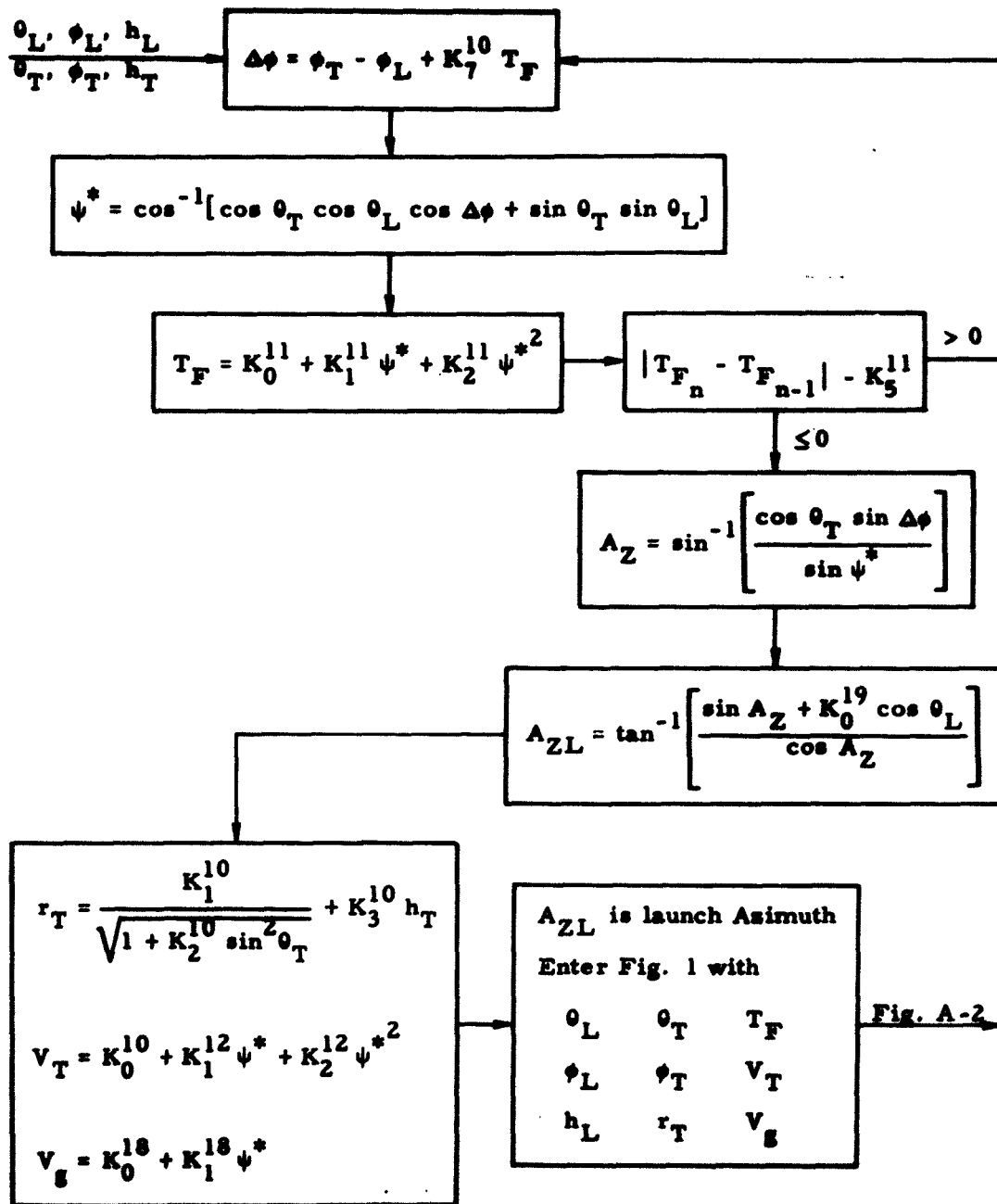


Figure A-1. Prelaunch Computations

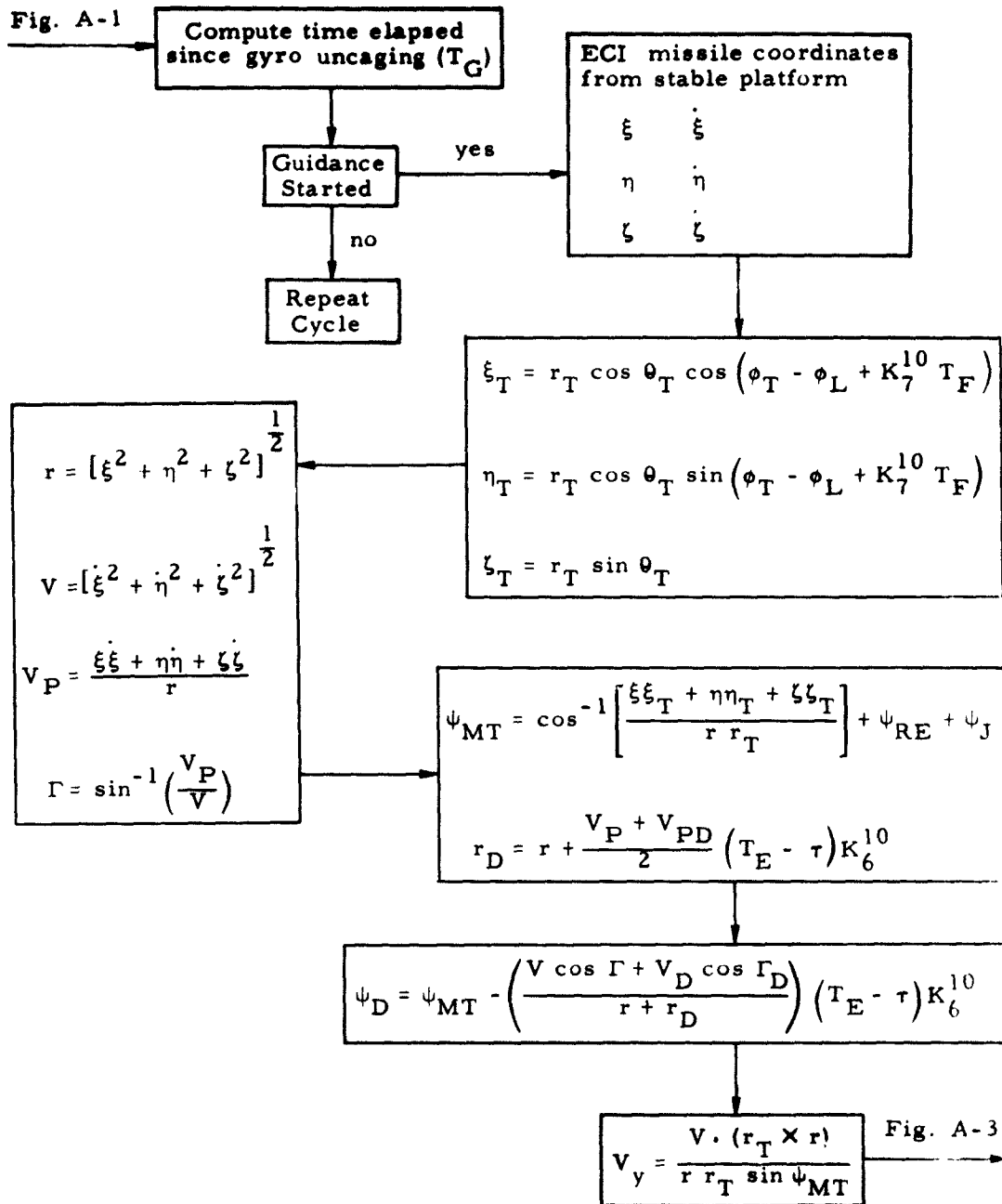


Figure A-2. Coordinate Transformation

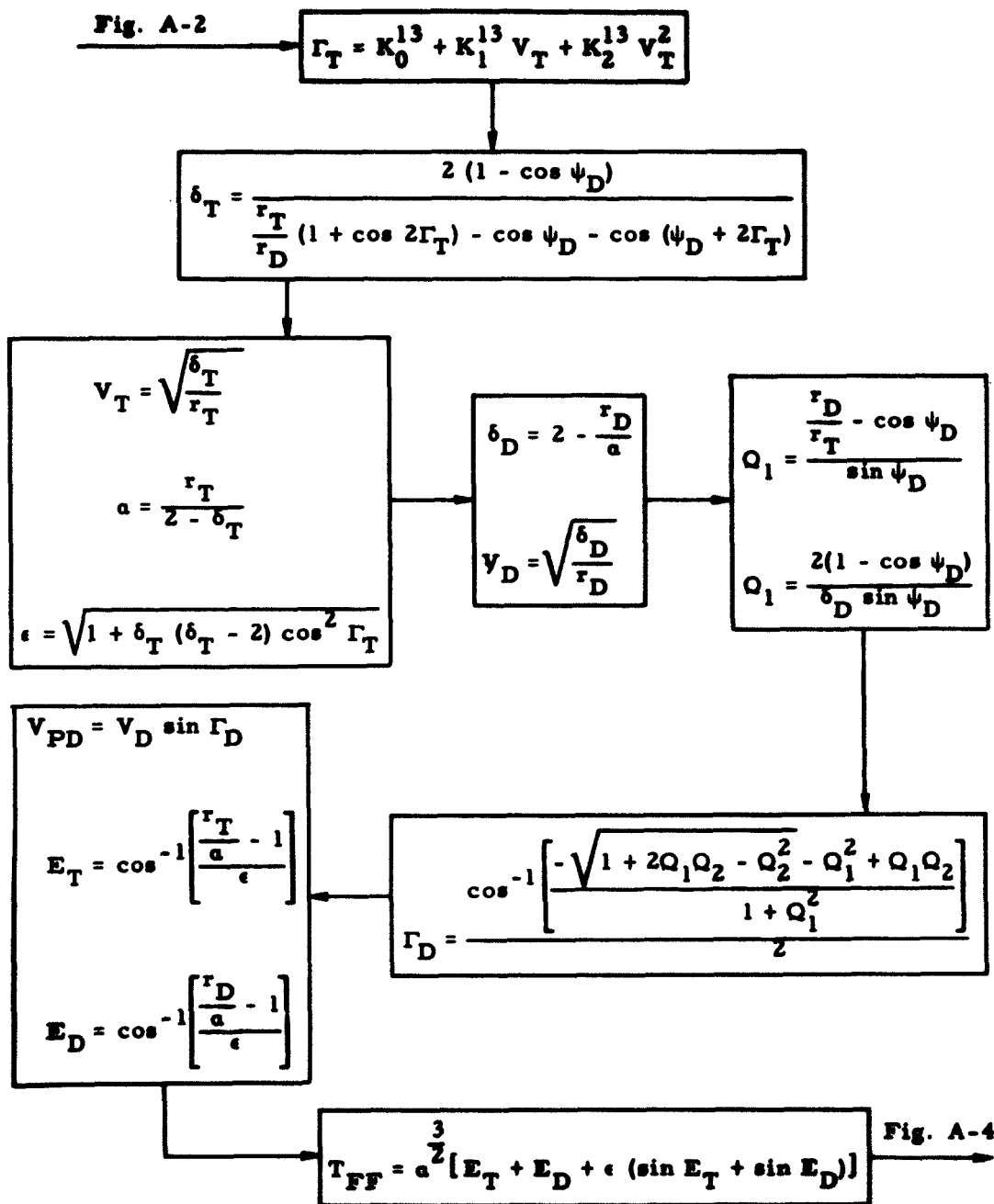


Figure A-3. Orbital Parameters

Fig. A-3

$$\begin{aligned}
Q_9 &= \sin A_Z \\
Q_{10} &= \sin \theta_L \\
Q_{11} &= \cos A_Z \cos \theta_L \\
Q_{12} &= Q_{10}^2 (2 + \cos \psi_D) + 2Q_{10}Q_{11} \sin \psi_D + Q_{11}^2 (1 - \cos \psi_D) \\
Q_{13} &= (Q_{10}^2 - Q_{11}^2) [K_0^{20} + K_1^{20} \psi^* + K_2^{20} \psi^{*2}] - 2Q_{10}Q_{11} [K_0^{21} + K_1^{21} \psi^* + K_2^{21} \psi^{*2}] \\
Q_{17} &= (K_0^{17} + K_1^{17} \theta_L) + (K_2^{17} + K_3^{17} \theta_L) \psi^* \\
Q_{16} &= \frac{1}{1 + \left[ \frac{\alpha}{r_T} (1 + \epsilon) - 1 \right] K_0^{16}}
\end{aligned}$$

$$\begin{aligned}
\psi_J &= \frac{2K_8^{10} (1 - \cos \psi_D) Q_{16}}{\delta_D^2 \sin 2\Gamma_D \cos^2 \Gamma_D} \left[ Q_{16} \delta_D \cos^2 \Gamma_D (1 - Q_{12} + Q_{13}) + \frac{2}{3} (Q_{12} - 3Q_{10}^2) \right] \\
V_{yD} &= - \frac{2K_8^{10} V_D Q_{17} Q_9 \cos \theta_L}{\delta_D \cos \Gamma_D \sin \psi_D} [Q_{10} (1 - \cos \psi_D) + Q_{11} (\psi_D - \sin \psi_D)]
\end{aligned}$$

$$\begin{aligned}
T_J &= (K_0^{23} + K_1^{23} \theta_L) + (K_2^{23} + K_3^{23} \theta_L) \psi^* + \\
&\quad (K_4^{23} + K_5^{23} \theta_L) \psi^{*2} - \psi^{*2} (K_6^{23} + K_7^{23} \theta_L) \sin |2A_Z|
\end{aligned}$$

$$\begin{aligned}
\psi_{RE} &= \sum_{n=0}^4 K_n^{22} V_T^n + h \sum_{n=0}^3 K_n^{24} V_T^n + h^2 \sum_{n=0}^3 K_n^{25} V_T^n \\
T_{RE} &= K_{20}^{14} + K_{21}^{14} h_T + K_{22}^{14} V_T
\end{aligned}$$

Fig. A-5

Figure A-4. Perturbation Corrections



Fig. A-4

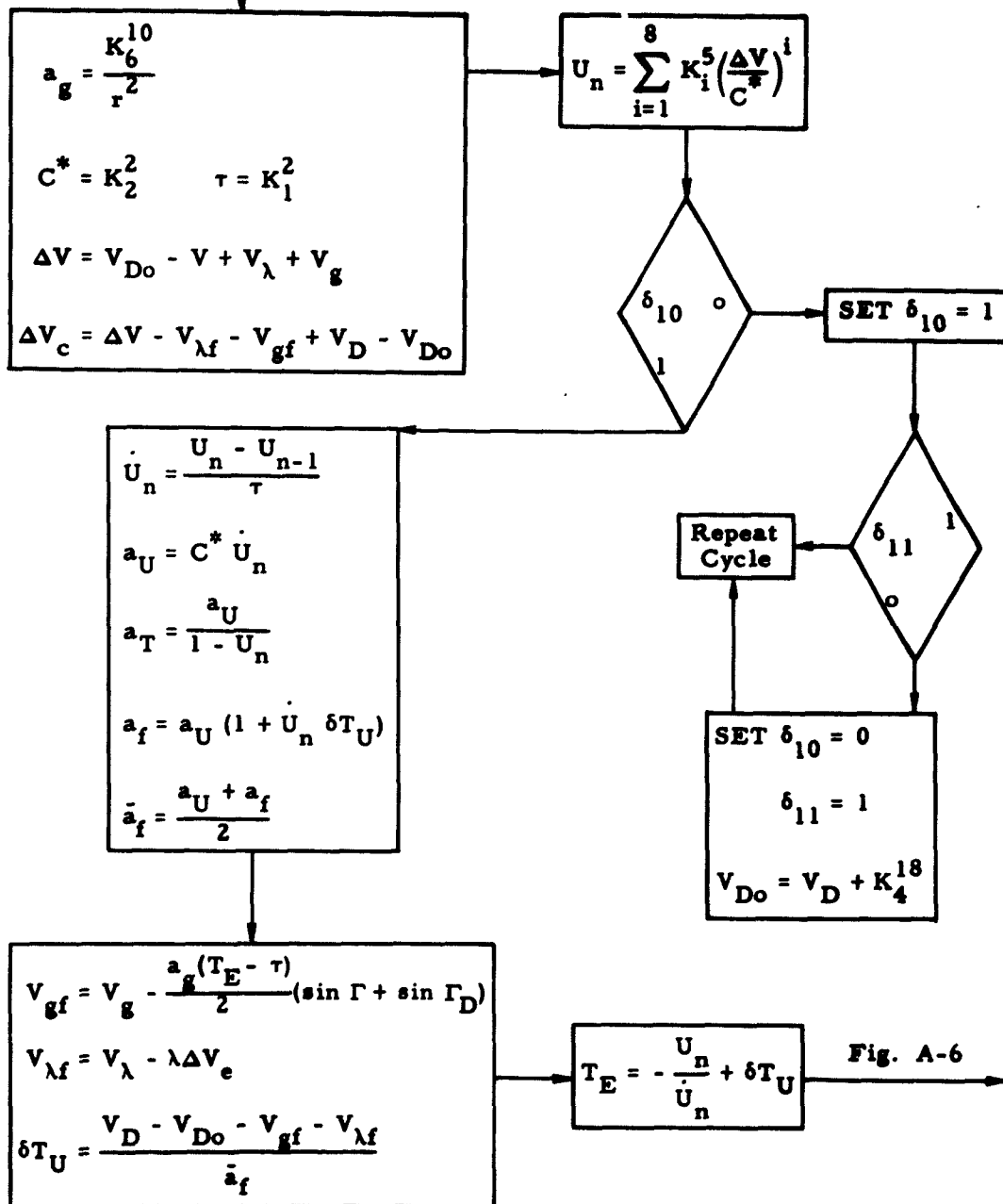


Figure A-5 Time-To-Go Computation

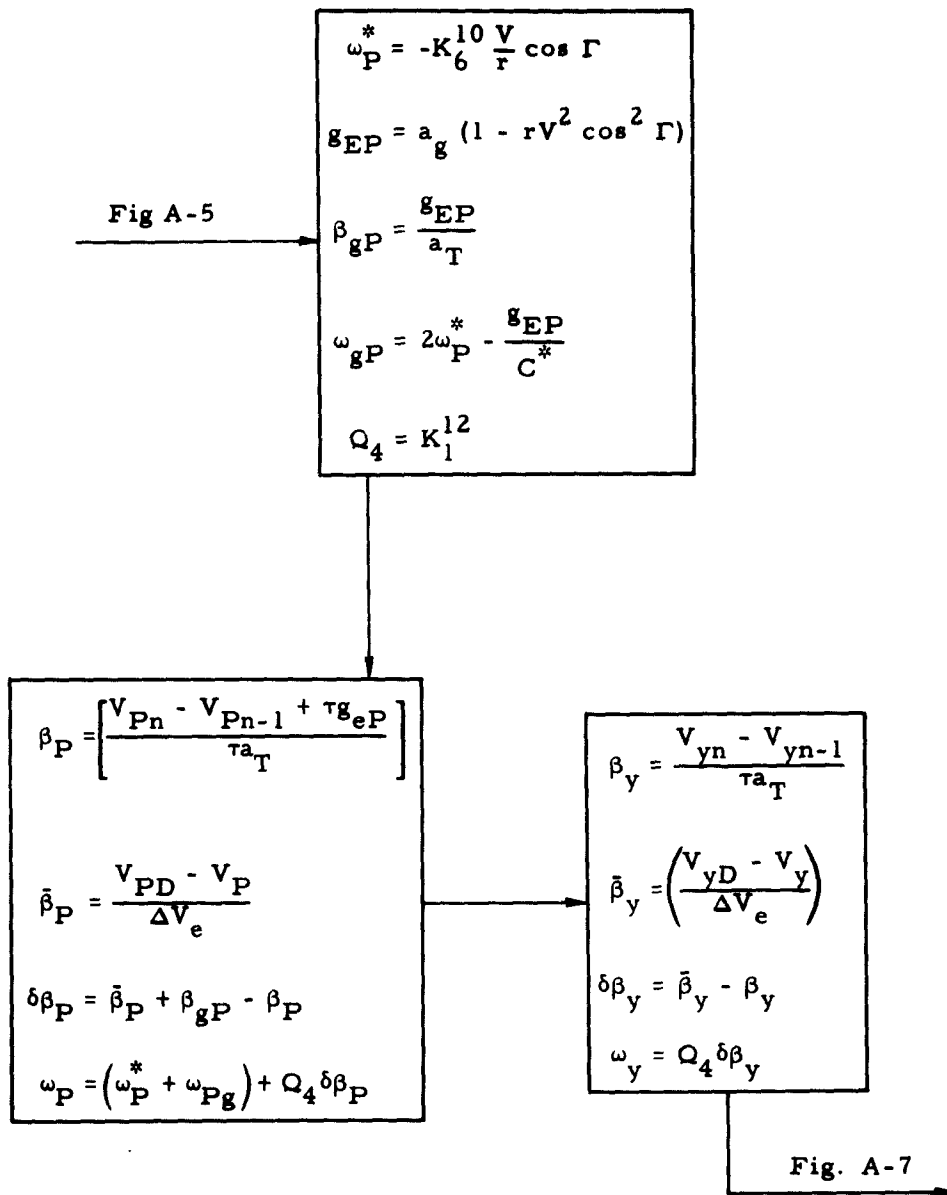


Figure A-6. Steering Commands

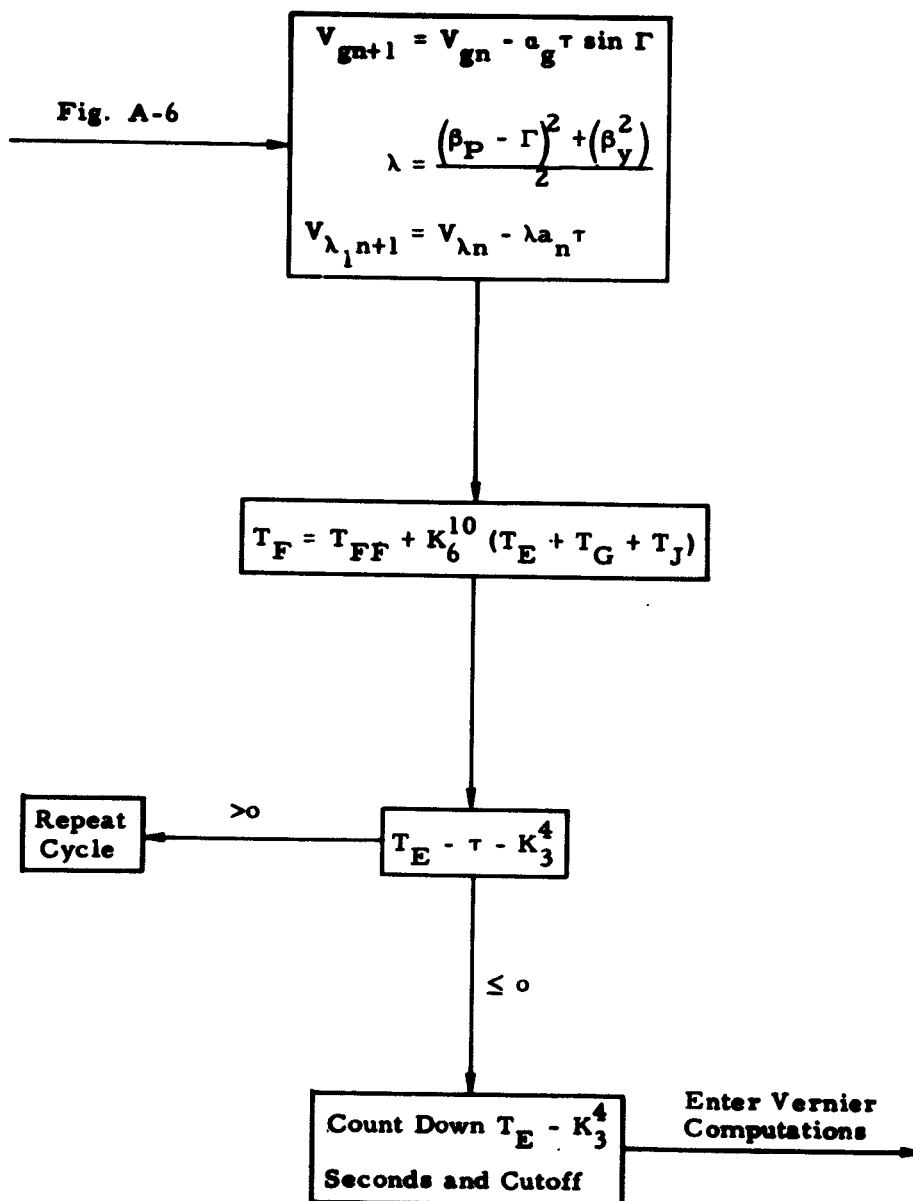


Figure A-7. Cutoff Computation

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<p>Aerospace Corporation, El Segundo, California.  <b>SELF-CONTAINED EXPLICIT GUIDANCE EQUATIONS FOR BALLISTIC MISSILES.</b>          Prepared by Duncan MacPherson.          March 1963. [40]p. incl. illus.          (Report TDR-169(3335)TN-2; BSD-TDR-63-41)          (Contract AF 04(695)-169) Unclassified report</p> <p>This document outlines a guidance equation mechanization for use with ballistic missiles. This technique does not require pre-launch targeting computations and is therefore especially suitable for mobile weapons systems.</p>	<p>UNCLASSIFIED</p>
<p>Aerospace Corporation, El Segundo, California.  <b>SELF-CONTAINED EXPLICIT GUIDANCE EQUATIONS FOR BALLISTIC MISSILES.</b>          Prepared by Duncan MacPherson.          March 1963. [40]p. incl. illus.          (Report TDR-169(3335)TN-2; BSD-TDR-63-41)          (Contract AF 04(695)-169) Unclassified report</p> <p>This document outlines a guidance equation mechanization for use with ballistic missiles. This technique does not require pre-launch targeting computations and is therefore especially suitable for mobile weapons systems.</p>	<p>UNCLASSIFIED</p>

<p>Aerospace Corporation, El Segundo, California.  <b>SELF-CONTAINED EXPLICIT GUIDANCE EQUATIONS FOR BALLISTIC MISSILES.</b>          Prepared by Duncan MacPherson.          March 1963. [40]p. incl. illus.          (Report TDR-169(3335)TN-2; BSD-TDR-63-41)          (Contract AF 04(695)-169) Unclassified report</p> <p>This document outlines a guidance equation mechanization for use with ballistic missiles. This technique does not require pre-launch targeting computations and is therefore especially suitable for mobile weapons systems.</p>	<p>UNCLASSIFIED</p>
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